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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2023

First Semester

Complementary Course—MATHEMATICS—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course to Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

[Prior to 2013 Admissions]

Time : Three Hours | Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of four questions carries a weight of 1.

I. 1 What is $\lim_{x \to \pi/2} x \sin x$.

2 What is $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$?

3 Write the ϵ–δ definition of the limit of a function.

4 Find $\frac{ds}{dt}$ if $S = \frac{t}{2t+1}$.

II. 5 If $y = x^2 \sin x$, find $\frac{dy}{dx}$.

6 What is the absolute maximum value of $\cos x$ on $[-\pi/2, \pi/2]$?

7 State first derivative theorem for local extreme values.

8 Define a monotonic increasing function.

III. 9 What are the critical points of the function whose derivative is $f'(x) = (x-1)(x+2)$?

10 If $f'(x) = 0$ at ecah point x of an open interval (a, b), then what can one say about that function.

11 Define partial derivative of a function $f(x, y)$ with respect of y at a point (x_0, y_0).

12 If $f(x, y) = x^3 + 3xy^2 + x - 1$, find $\frac{\partial f}{\partial x}$.

Turn over

IV. 13 Find $\frac{\partial^2 f}{\partial y\,\partial x}$ if $f(x, y) = ye^x$.

14 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = \log_y x$.

15 Define cot hx in terms of e^x.

16 What is the relation between tan (ix) and tan hx.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question carries a weight of 1.

17 Does $\lim_{x \to 0} \frac{x}{|x|}$ exist. Justify your answer.

18 Find $\lim_{\theta \to 0} \frac{\sin \theta}{\sin^2 \theta}$.

19 Find the absolute maximum and minimum values of $f(x) = -x - 4$ on $-4 \le x \le 1$.

20 Find the function $f(x)$ whose derivative is sin x and whose graph passes through the point (0, 2).

21 For the function $g(t) = -3t^2 + 9t + 5$ find the intervals on which the function is increasing and decreasing.

22 Find $\frac{\partial z}{\partial x}$, given that $yz - \ln z = x + y$, where z is a function of two independent variables x and y.

23 Separate into real and imaginary parts sin $(\alpha - i\beta)$.

24 Show that $\cosh 2x = \cosh^2 x + \sinh^2 x$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question carries a weight of 2.

25 At time t, the position of a body moving along the S-axis is $S = t^3 - 6t^2 + 9t$ m. (a) Find the body's acceleration each time the velocity is zero (b) Find the body's speed each time the acceleration is zero.

26 Find the slope of the tangent to the curve $y = \sin^5 x$ at the point where $x = \pi/3$.

27 What is the largest possible area of a right triangle whose hypotensive is 5 cm. long.

28 Show that the function $f(x) = x^4 + 3x + 1$ has exactly one zero in in the interval $[-2, -1]$.

29 Verify that $wxy = wyx$ where $w = x\sin y + y\sin x + xy$.

30 Expand $\cos^5\theta$ in a series of cosines of multiples of θ.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question carries a weight of 4.

31 (a) If $2 - x^2 \le g(x) \le 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

(b) Find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$.

32 (a) Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$.

(b) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s, y = r + s$.

33 (a) Express $\sin^5 x \cos x$ in terms of sines of multiples of x.

(b) Sum to infinity the series $\cos x \sin x + \frac{1}{2!}\cos^2 x \sin 2x + \frac{1}{3!}\cos^3 x \sin 3x + \ldots$.

$(2 \times 4 = 8)$